Homework 4

Chung-Chia Kuo (B05703100)†, Yu-Chieh Kuo (B07611039)†, and Che Cheng (B07901012)‡

†Department of Information Management, National Taiwan University
‡Department of Electrical Engineering, National Taiwan University

PART I
Problems

Example 1: Screening Strategy

In this problem, we introduce a screening strategy widely adopted in the retail industry — using coupons to separating consumer's type. Retailers distribute cents-off coupons to consumers, and consumers decide whether to keep the coupon, and later whether to buy from the retailer. In particular, there's one kind of coupon offered after each purchase which cannot be used immediately. For instance, fast food restaurants such as Burger King, KFC and McDonald's have all given away such coupons.

The players involved are retailer and a group of heterogeneous consumers, in the role of principle and the agents respectively. The retailer first distributes coupons to consumers, and consumers decide whether to keep it or throw it away. Different consumer may have different holding cost. The consumers then have to decide whether they will consume at the retailer based on the price (with or without coupon) and gained utility. Note that when deciding on whether to consume again, the holding cost is sunk and not taken into consideration.

The consumers face a trade-off between costs of holding the coupons and the reduction in price, and thus act differently based on his/her type. By providing the coupon/no-coupon option, the retailer may charge more from the consumers with smaller price sensitivity who may find it too costly to hold the coupon, and still serve those that are sensitive to price.

In our opinion, one thing clever is the constraint that the coupon cannot be used right away. It not only encourages consumers to revisit, but also make the difference in holding cost significant, thus preventing the price-insensitive type from using it. We think the strategy screens consumers quite effectively, and can thus increase the revenue significantly.

Example 2: Signaling Strategy

In this problem, we introduce a signaling strategy adopted by students applying for collage in the US. To apply for any program in the US, a letter of recommendation (LoR) is mandatory, and the student has access to the LoR unless he chooses to waive the right. Whether the LoR is kept confidential is known to the institution to which the student is applying.

The game involves three players, the student, the professor, and the institution. The student first decides whether to waive his right or not, and ask the professor for a LoR. The professor, based on the student's decision and his impression about the student, then decided whether to write one for him and what to write. Finally, the institution grade the student based on the content of the LoR and whether the student has waived his right.

Even if the professor is not affected by the student's choice, the institution may not always believe so, and would possibly put less weight on the LoR if it is not confidential. By waiving his access to the LoR, the student may signal that the content of the LoR is more trustworthy. However, if the student does not trust the professor or is unaware of the signaling effect, he might want to read the LoR.

We believe that the signal does work because some students do not trust the professor, and that the institution does not trust the professor. The former results in some students sending different signal, and the latter results in the institution putting less weight on the LoRs that are available to students.

PART II

Case Study

Section 1: Research Topic and Assumptions

We follow from the hypothesis in HW2 that providing different payment options would larger the demand, as consumers have heterogeneous and *horizontal* difference in preference for different payment methods. To simplify the discussion, we assume it is publicly known that the retailer setting up a platform on his own is too costly and thus never an option. The platform, on the other hand, already has a working service, and the setup cost is sunk and thus not in consideration.

We keep the assumptions that unit production cost is 0, and that the platform has all bargaining power over the retailer. We also keep the assumption that the retailer must set the same price for different payment methods, and discuss its effect qualitatively. The main difference is that we now assume the *quality* of the platform, i.e. the potential demand it is bringing is privately known to the platform. In addition, we assume that the maintenance cost of the platform is 0, as it has little to do with the topic in consideration. This assumption directly leads to the fact that both types are efficient.

We aim to model the situation as a signaling game, and solve for the conditions for which a separating equilibrium exists.

Section 2: The Model

The model considers a mobile payment service provider (called the platform in the sequel), a retailer and a group of heterogeneous consumers. The retailer is selling one product only with unit production cost 0. Each consumer is endowed with $x \in R$, and they lie uniformly with density 1. Each of them obtain a utility of $V - p$ if getting the product at price p. However, consumer x must incur a mental cost of $|x|$ if he pays by cash due to inconvenience.

The platform is endowed with a mobile payment platform characterized by either X_H or X_L privately known to the platform, with $X_H > X_L > 0$, where consumer x incurs a mental cost of $|x - X_i|$ if he pays using the service provided by type-i platform. The retailer has prior belief that the platform is of type-L with probability β and type-H with probability $1 - \beta$[1].

If the retailer joins the platform, he has to pay a fixed fee f to the platform as well as ϕp for each product paid by mobile payment.

The game proceeds as follows. First, the platform offers a contract (f, ϕ) to the retailer. Given the contract, the retailer decides whether to accept the contract and join the platform or keep on accepting cash only. The retailer then announces the retail price p, and each consumer buys a product with the payment method generating the highest nonnegative utility for him, or nothing if all payment methods generates a negative utility.

Section 3: Summary of Previous Results

The results without information asymmetry is derived in HW2, and below summarized some important results that are used in this research.

First, the reservation value (the payoff generated when accepting cash only) of the retailer is $\frac{V^2}{2}$. After

[1] To ease notation, i is used to denote the type of a platform when speaking in general, and it is assumed that $i \in \{H, L\}$ whenever not specified otherwise.

accepting a contract (f_i, ϕ_i), the retailer chooses

$$p_i^* = \begin{cases} \frac{V}{2} + \frac{X_i}{4} & X_i \leq \frac{2V}{3} \\ V - \frac{X_i}{2} & \frac{2V}{3} < X_i < V \\ \frac{V}{2} & X_i \geq V\,, \end{cases}$$

and generates payoff

$$\pi_i^* = \begin{cases} \frac{(2-\phi_i)(2V+X_i)^2}{16} - f_i & X_i \leq \frac{2V}{3} \\ (2-\phi_i)X_i\left(V - \frac{X_i}{2}\right) - f_i & \frac{2V}{3} < X_i < V \\ \frac{(2-\phi_i)V^2}{2} - f_i & X_i \geq V\,. \end{cases}$$

We define the *efficiency* of type-i platform as the extra total profit of introducing mobile payment, which is

$$\eta_i \coloneqq \begin{cases} \frac{X_i(4V+X_i)}{8} & X_i \leq \frac{2V}{3} \\ \frac{V^2}{2} - (V-X_i)^2 & \frac{2V}{3} < X_i < V \\ \frac{V^2}{2} & X_i \geq V\,. \end{cases}$$

As the platform can extract all extra profit from the retailer, both platforms will always be introduced as the maintenance cost C is set to 0 in the current model.

Since f and ϕ are both neutral, in a sense that they do not affect the pricing strategy of the retailer, the equilibrium contract can be anything satisfying $\pi_i^* = \frac{V^2}{2}$, so that the retailer is indifferent between accepting or not. That is, (f_i, ϕ_i) is such that

$$f_i = \begin{cases} \frac{2X_i(4V+X_i) - \phi_i(2V+X_i)^2}{16} & X_i \leq \frac{2V}{3} \\ (2-\phi_i)X_i\left(V - \frac{X_i}{2}\right) - \frac{V^2}{2} & \frac{2V}{3} < X_i < V \\ \frac{(1-\phi_i)V^2}{2} & X_i \geq V \end{cases}$$

and $\phi_i \in [0,1]$. We will refer to any contract satisfying the above condition as a first-best contract for type-i, denoted $(f_i^{\mathrm{FB}}, \phi_i^{\mathrm{FB}})$.

The demand for type-L mobile payment with retail price p is

$$D_L = \begin{cases} V - p + \frac{X_L}{2} & p \leq V - \frac{X_L}{2} \\ 2(V-p) & V - \frac{X_L}{2} < p < V \\ 0 & p \geq V\,, \end{cases}$$

which will later be used to derive the profit earned by type-L platform when deviating.

Finally, observe that $X_L \geq V$ makes the retailer indifferent between joining which one, thus we focus only on cases in which $X_L < V$.

Section 4: Model Solving

In order to support the separating equilibrium, we assume that except when seeing the equilibrium contract (f_H, ϕ_H) for type-H platform, the retailer will always believe that the signal comes from type-L platform with probability 1. In such case, type-L platform only need to choose between adopting his first-best contract $(f_L^{\mathrm{FB}}, \phi_L^{\mathrm{FB}})$ or the type-H contract (f_H, ϕ_H), since all other options are dominated by the first-best one.

We shall next derive some useful properties to simplify the derivation. We use the notation $\sim_i, \succeq_i, \succ_i$ to denote that type-i is indifferent, weakly prefers and strictly prefers a contract to another, and $\prec_i, \preceq_i$ analogously.

Proposition 1. *Suppose the retailer believes that the platform is of type-H upon seeing* (f_1, ϕ_1) *or* (f_2, ϕ_2), *where* $0 \leq \phi_1 < \phi_2 \leq 1$. *Under such belief, if* $(f_1, \phi_1) \sim_H (f_2, \phi_2)$, *then* $(f_1, \phi_1) \succeq_L (f_2, \phi_2)$.

Proof. Let r_i denote the total revenue earned by mobile payment when the platform is correctly identified as type-i, and let r_{LH} denote the revenue earned by mobile payment when type-L platform is misbelieved to be of type-H[2].

We first argue that $r_H \geq r_{LH}$. Observe that $r_i = \frac{\pi_i^*|_{f_i=\phi_i=0}}{2}$, where π_i^* is the payoff for the retailer if he accepts a contract (f_i, ϕ_i) from type-i, as shown in Section 3. We can reinterpret the total revenue r_i as a function $r(X)$, and it is easy to verify that r is continuous and has nonnegative first-order derivative, i.e. it is increasing in X. Thus, we have $r_H \geq r_L$.

Note next that the retail price p is set to the system optimal value according to the type believed by the retailer, and thus any the revenue generated by setting $p = p_H^*$ when in fact the platform is of type-L may not generate a higher total revenue, i.e. $r_{LH} \leq r_L$. Combing the above gives $r_H \geq r_L \geq r_{LH}$.

Finally, we have

$$\begin{aligned} f_2 + \phi_2 r_{LH} &= (f_1 + \phi_1 r_H - \phi_2 r_H) + \phi_2 r_{LH} \\ &= f_1 + \phi_1 r_{LH} + (\phi_2 - \phi_1)(r_{LH} - r_H) \\ &\leq f_1 + \phi_1 r_{LH}\,, \end{aligned}$$

i.e. $(f_1, \phi_1) \succeq_L (f_2, \phi_2)$. □

Proposition 1 leads directly to the following corollary:

Corollary 1. *Given any* (f_1, ϕ_1) *that can successfully serve as a type-H signal in a separating equilibrium, there exists some* $(f_2, 1)$ *such that*

1. $(f_1, \phi_1) \sim_H (f_2, 1)$, *and*
2. $(f_2, 1)$ *can serve as a type-H signal as well.*

Therefore, it suffices to restrict to contract with $\phi = 1$. Recall that in the first-best scenario, platform-i earns exactly η_i. We reinterpret the first-best condition

$$f_i = \begin{cases} \dfrac{2X_i(4V + X_i) - \phi_i(2V + X_i)^2}{16} & X_i \leq \dfrac{2V}{3} \\ (2 - \phi_i)X_i\left(V - \dfrac{X_i}{2}\right) - \dfrac{V^2}{2} & \dfrac{2V}{3} < X_i < V \\ \dfrac{(1 - \phi_i)V^2}{2} & X_i \geq V \end{cases}$$

as a function $f^{\mathrm{FB}}(X, \phi)$, which denotes the first-best transfer payment a platform characterized by X charges when the revenue sharing ratio is ϕ. An interesting observation here is that $f^{\mathrm{FB}}(X, 1)$ is continuous and $\frac{\partial f^{\mathrm{FB}}}{\partial X}|_{\phi=1} \geq 0$, i.e. $f^{\mathrm{FB}}(X, 1)$ is increasing in X. We shall further discuss this in Section 5.

Using the condition that IC constraint must hold, i.e. $\pi_{LH} \leq \pi_L^{\mathrm{FB}}$, we can derive an upper bound for f_H, which is essentially $\overline{f} = \eta_L - r_{LH}$ by plugging $\phi = 1$ into the IC constraint. The following discussion utilize the demand D_L summarized in Section 3.

First consider the case where $X_H \geq V$, i.e. $p_H^* = \frac{V}{2}$. In such case, the total revenue of mobile payment is

$$\begin{aligned} r_{LH} &= p_H^* \cdot D_L(p_H^*) \\ &= \frac{V}{2} \cdot \frac{V + X_L}{2} \\ &= \frac{V(V + X_L)}{4}\,. \end{aligned}$$

Next consider the case where $\frac{2V}{3} < X_H < V$, i.e. $p_H^* = V - \frac{X_H}{2}$. In such case, the total revenue of mobile payment is

$$\begin{aligned} r_{LH} &= p_H^* \cdot D_L(p_H^*) \\ &= \left(V - \frac{X_H}{2}\right) \cdot \frac{X_H + X_L}{2} \\ &= \frac{(2V - X_H)(X_H + X_L)}{4}\,. \end{aligned}$$

[2]Note that these r's depend only on X_H and/or X_L, and are the same regardless of the contract, since it has been shown that both f and ϕ do not affect the retail price.

Finally, suppose $X_H \leq \frac{2V}{3}$, i.e. $p_H^* = \frac{V}{2} + \frac{X_H}{4}$. In such case, the total revenue of mobile payment is

$$\begin{aligned} r_{LH} &= p_H^* \cdot D_L(p_H^*) \\ &= \left(\frac{V}{2} + \frac{X_H}{4}\right) \cdot \left(\frac{V}{2} - \frac{X_H}{4} + \frac{X_L}{2}\right) \\ &= \frac{(2V + X_H)(2V + 2X_L - X_H)}{16} . \end{aligned}$$

Thus, we have

$$\overline{f} = \begin{cases} \eta_L - \dfrac{(2V + X_H)(2V + 2X_L - X_H)}{16} & X_H \leq \dfrac{2V}{3} \\ \eta_L - \dfrac{(2V - X_H)(X_H + X_L)}{4} & \dfrac{2V}{3} < X_H < V \\ \eta_L - \dfrac{V(V + X_L)}{4} & X_H \geq V . \end{cases}$$

Costless signaling is possible if and only if $f^{\mathrm{FB}}(X_H, 1) \leq \overline{f}$. Define

$$\begin{aligned} L(X_H, X_L) &= f^{\mathrm{FB}}(X_H, 1) - \overline{f} \\ &= \begin{cases} \dfrac{2X_H(4V + X_H) - (2V + X_H)^2}{16} - \eta_L + \dfrac{(2V + X_H)(2V + 2X_L - X_H)}{16} & X_H \leq \dfrac{2V}{3} \\ X_H\left(V - \dfrac{X_H}{2}\right) - \dfrac{V^2}{2} - \eta_L + \dfrac{(2V - X_H)(X_H + X_L)}{4} & \dfrac{2V}{3} < X_H < V \\ 0 - \eta_L + \dfrac{V(V + X_L)}{4} & X_H \geq V . \end{cases} \end{aligned}$$

It can be seen that L is continuous both in X_H and X_L, and $\frac{\partial L}{\partial X_H} \geq 0$ and $\frac{\partial L}{\partial X_L} \leq 0$. This means that L is increasing in X_H. Thus, it suffices to show that $L(X_H, 0) \geq 0$ for $X_H \leq \frac{2V}{3}$, which is essentially $4VX_H \geq 0$. Thus, we conclude that costless signaling is never possible, and the equilibrium type-H contract is $(f_H, \phi_H) = (\overline{f}, 1)$.

Section 5: Results and Discussions

The major result is that costless signaling is impossible in the proposed model. The type-H platform must lower its earnings to prevent himself from being mimicked. This research also gives a potential explanation to why revenue sharing is commonly seen in real world situations, as it can be used as an signaling instrument.

We next discuss some interesting observations. First, the separating equilibrium is efficient for the whole system, i.e. information asymmetry doesn't harm the society. It only prevent the type-H platform from extracting all revenue from the retailer. This, as discussed in HW2, is due to the assumption that the price for both payment methods must be the same. If such was not the case, then problem of moral hazard will occur and makes the system inefficient whenever $\phi > 0$, thus making the whole system inefficient in the second-best scenario.

The pros and cons for type-L to deviate is also quite interesting. The biggest benefit is essentially that he can deceive the retailer to accept a extremely high f, and thus earns more. However, since $r_{LH} \leq r_L^{\mathrm{FB}}$, as shown in Proposition 1 as an intermediate result, the system inefficiency caused by the retailer's misbelief of X hurts the type-L platform. Although not studied, it is expected that similar inefficiency also occur in a pooling equilibrium, unless renegotiation is allowed.

Finally, Note that $f^{\mathrm{FB}}(X, 1)$ is increasing in X, i.e. even when $\phi = 1$, the retailer still earns more when X is higher. This is the main reason that costless signaling is impossible, since $f^{\mathrm{FB}}(X_H, 1) \geq f^{\mathrm{FB}}(X_L, 1)$ always holds.